

Mechanical system reliability analysis using reliability matrix method

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Abstract. Mechanical systems, including building ones, consist of a huge number of elements. Thus, traditional methods of reliability block diagram and logical schemes, oriented to small-element systems, aren't appropriate for reliability analysis. In this paper a fundamentally new method for reliability assessment of mechanical systems is considered. It's based on the reliability matrix. The reliability matrix is the difference between the characteristic matrix and the connection matrix, which are formed from the structural scheme. Probability of failure or probability of success of the system is equal to determinant of the reliability matrix. The use of the matrix method proposed by the authors in conjunction with computer software has more perspective, it allows to work with complex mechanical systems, significantly reduces the time of analysis and decrease the risk of errors. This method of calculation is optimized for inclusion in Civil Engineering CAD software.

1. Introduction

Reliability is a property of objects to keep in time ability to performance of the required functions in the set of specified modes and conditions, maintenance of use, storage and transportation [1]. Here the term “objects” is the most general name of mechanical system [2]. The mechanical system is a holistic set of the interconnected elements separated from the environment. Systems are simple if they can be presented as the chain of sequentially or parallel connected elements. Refusal at least one sequentially connected element or all parallel connected elements leads to a system's fail. Complex systems are characterized by a combined structure, when the subsystem (and its reliability) can be considered separately from basic system [1, 3]. In this paper a complex system which elements' reliability is defined in advance and is an independent event are considered. Any building or structure can be presented in the same way. For instance, an industrial building can be represented as a set of cross frames, stiffening frames and cover stiffening diaphragm [4].

Reliability block diagram is a widely employed method of system reliability analysis. It's used if it is possible to split the complex system into subsystems, for each of which the probability of success or failure can be defined. In operating process, the object can be only in a workable or failure state. Thus the element is characterized by the probability of success p_i or the probability of failure q_i , connected by the following expression [5, 6]:



$$p_i + q_i = 1. \quad (1)$$

There are various methods based on the application of structural schemes. These include methods based on the formula of total probability, using of truth tables, Boolean functions, logic-probability method, method of binary decision diagrams, as well as the matrix method. [6, 7, 8, 9, 10].

At the present stage of scientific and technical progress matrixes are applied everywhere. They can describe the equation of motion of rigid body [11], internal force in statically determinate and indeterminate systems can be calculated with matrix application (Finite Element Analysis) [12]. Therefore it is a great importance to have a use-friendly method of matrix determination of system reliability.

2. Basic concept of assessing system reliability using the reliability matrix

The existing matrix reliability analysis method is based on the logical-probabilistic method where Boolean functions are used to express analytic conditions of systems operability and their strict interconnections with probability functions. A matrix of incompatible states is formed to enumerate all possible states of the system. Each state in the matrix is checked by the logical condition for operability [9]. This method is not wide-spread, since it is quite laborious and inconvenient method even with the matrix application.

The method based on graph theory and Boolean function, described by J. Tang, also uses matrixes [8]. This method was developed in the Russian scientist's works [6]. It is less laborious than the existing matrix method, it has a great potential for implementing the algorithm method in software systems, but it also has a number of deficiencies. It's the method what system reliability analysis using the reliability matrix is based on.

2.1. Basic provisions of method based on graph theory and Boolean function

According to the method based on graph theory and Boolean function mechanical system is represented as a graph, its elements are points of the graph, and the connection between the elements is a link indicated a joining pair. The reliability block diagram of arbitrary mechanical system is shown in figure 1, and it's graph is shown in figure 2.

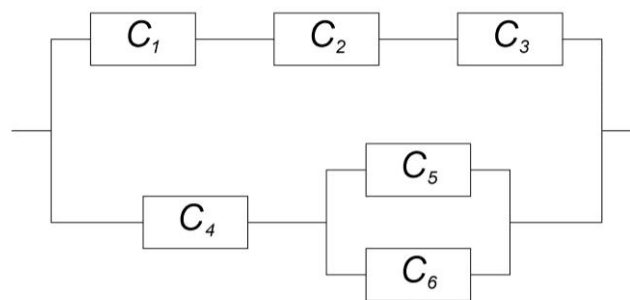


Figure 1. Reliability block diagram of a mechanical system

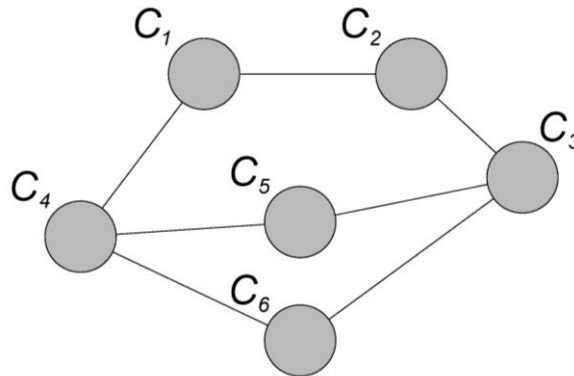


Figure 2. System graph

To have a completed expression for system reliability, three matrices are introduced, which are the component connection matrix Ω , the component reliability matrix R , and the system reliability matrix Γ .

The component reliability matrix R of system made up of n components is diagonal and defined as follows:

$$R = \begin{pmatrix} p_1 & 0 & \dots & 0 \\ 0 & p_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & p_n \end{pmatrix}. \quad (2)$$

The component connection matrix Ω is defined by replacing the non-zero elements of the adjacency matrix by the corresponding values of the interconnection between elements obtained by taking the expectation of the path:

$$\omega_{ij} = \prod_{k=i}^j [(1-p_k)\overline{x_k} \vee p_k x_k], i < j, ij \notin Path. \quad (3)$$

The system reliability matrix Γ of the system is a diagonal matrix and defined by the following expression:

$$\Gamma = R - \Omega. \quad (4)$$

And then system reliability is calculated as a determinant of system reliability matrix

$$P_s = \det(\Gamma) \quad (5)$$

For example, consider a mechanical system represented in figure 1, component reliability is shown in table 1. According to this method system reliability evaluation is $P_s = 0.885483$ [6, 8].

Table 1. The component reliability of the system

Component	C_1	C_2	C_3	C_4	C_5	C_6
Reliability	0.98	0.98	0.98	0.96	0.99	0.99

Also the reliability of this system is evaluated directly by the following expression:

$$P_s = 1 - (1 - p_1 p_2 p_3) \cdot \{1 - p_4 \cdot [1 - (1 - p_5) \cdot (1 - p_6)]\} = 0.997642. \quad (6)$$

3. Basic concept of assessing system reliability using the reliability matrix

Operate with simple structures of components to eliminate difference between the results.

Consider a set of n elements connected in series, shown in figure 3. The component reliability matrix R is written in expression (2).

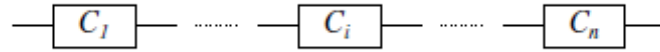


Figure 3. Reliability block diagram of set of series connected elements

Then component connection matrix according to paper [8] is expressed by follows:

$$\Omega = \begin{pmatrix} 0 & \omega_{12} & 0 & \dots & 0 \\ \omega_{12} & 0 & \omega_{23} & \dots & \dots \\ 0 & \omega_{23} & 0 & \dots & 0 \\ \dots & \dots & \dots & 0 & \omega_{(n-1)n} \\ 0 & \dots & 0 & \omega_{(n-1)n} & 0 \end{pmatrix}. \quad (7)$$

The system reliability matrix Γ is as follows:

$$\Gamma = \begin{pmatrix} p_1 & -\omega_{12} & 0 & \dots & 0 \\ -\omega_{12} & p_2 & -\omega_{23} & \dots & \dots \\ 0 & -\omega_{23} & p_3 & \dots & 0 \\ \dots & \dots & \dots & p_{(n-1)} & -\omega_{(n-1)n} \\ 0 & \dots & 0 & -\omega_{(n-1)n} & p_n \end{pmatrix}. \quad (8)$$

Obviously, that $P_s = \det(\Gamma) \neq \prod_{i=1}^n p_i$.

Similarly forms the system reliability matrix for set of n elements connected in parallel is shown in figure 4.

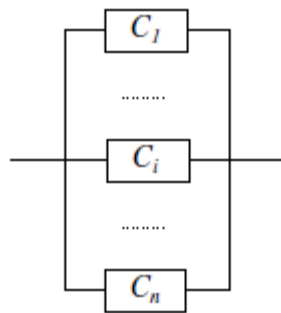


Figure 4. Reliability block diagram of set of parallel connected elements

The component connection matrix Ω is zero, then the system reliability matrix Γ will coincide with the component reliability matrix R :

$$\Gamma = R = \begin{pmatrix} p_1 & 0 & 0 & \dots & 0 \\ 0 & p_2 & 0 & \dots & \dots \\ 0 & 0 & \dots & \dots & 0 \\ \dots & \dots & \dots & p_{n-1} & 0 \\ 0 & \dots & 0 & 0 & p_n \end{pmatrix}. \quad (9)$$

Also obviously, that $P_s = \det(\Gamma) = \prod_{i=1}^n p_i \neq \prod_{i=1}^n (1 - p_i)$.

During the analysis, it was noted that the reliability matrix Γ_1 of a system of series connected elements (figure 3) can be represented as a diagonal matrix, where the elements of the main diagonal are the probabilities of failure-free operation of the elements.

$$\Gamma_1 = \begin{pmatrix} p_1 & 0 & 0 & \dots & 0 \\ 0 & p_2 & 0 & \dots & \dots \\ 0 & 0 & \dots & \dots & 0 \\ \dots & \dots & \dots & p_{n-1} & 0 \\ 0 & \dots & 0 & 0 & p_n \end{pmatrix}. \quad (10)$$

Then the reliability of the system of n series-connected elements is determined as follows:

$$P_1 = \det(\Gamma_1) = \prod_{i=1}^n p_i. \quad (11)$$

Similarly it is possible to form a failure matrix Γ_2 of a system of n parallel connected elements (figure 4), where the probabilities of failures of the elements will be located on the main diagonal.

$$\Gamma_2 = \begin{pmatrix} q_1 & 0 & 0 & \dots & 0 \\ 0 & q_2 & 0 & \dots & \dots \\ 0 & 0 & \dots & \dots & 0 \\ \dots & \dots & \dots & q_{n-1} & 0 \\ 0 & \dots & 0 & 0 & q_n \end{pmatrix}. \quad (12)$$

Then the failure probability of the system of series-connected elements is determined as follows:

$$Q_2 = \det(\Gamma_2) = \prod_{i=1}^n q_i. \quad (13)$$

3.1. Basic provisions of new matrix method

In the new method, it's assumed that the elements and set of elements connected in series are characterized by the value of reliability, and the elements and systems of elements connected in parallel are characterized by the value of the probability of failure. Thus under reliability analysis, complex systems should be reduced to simple ones by replacing subsystems into elements with the same probabilities of fail-free operation or failure.

However, for complex systems, it will be difficult to repeat such a procedure for all subsystems. Therefore, according to the ideas of J. Tang [7], the system reliability matrix Γ can be represent in the form of a difference between the component reliability matrix R and the component connection matrix Ω .

These matrices are determined on the basis of the reliability block diagram.

The components characteristics matrix R is formed in such a way that the values of probability of failure or reliability of elements are put on the matrix main diagonal. The values of q_i and p_i are determined by the following algorithm:

- The reliability block diagram of mechanical system is analyzed, simple elements and subsystems are distinguished. Subsystems are considered as separate systems, as well as elements and subsystems from their structures are distinguished until only simple elements remain;

- All simple elements are associated with the reliability if they are connected in series with the elements of their subsystem, and with the probability of failure if they are connected in parallel with the elements of their subsystem.

The relationships matrix Ω is formed as follows:

- If elements C_i and C_j ($j=i+1$) are connected in series, then -1 is placed in the cell ω_{ij} , if they are connected in parallel, then 1 is placed in the cell ω_{ij} (matrix is filled above the main diagonal);
- If elements C_i and C_j ($j \neq i+1$) are initial and final element of the subsystem of identically connected elements, then -1 is placed in the cell ω_{ji} , (matrix is filled below the main diagonal). But -1 is not placed if the given chain of elements represents whole system under consideration;
- If in the chain of elements there are complex elements (for example, a serial connection of parallel connected elements chain), then the above rules extend to the final element of the initial subsystem and to the initial element (if the matrix is filled above the main diagonal) and to the final element (if the matrix is filled below the main diagonal) of the next subsystem.

It should be mentioned, that the initial system is ultimately presented as a set of series connected elements; the matrix Γ determinant module is interpreted as reliability of the system, otherwise in case of parallel connection it is interpreted as the probability of failure.

There are restrictions for using the method:

- A mechanical system is considered as a hierarchical system with a finite number of subsystems, definite structure and scheme of interconnections. The scheme of interconnections and the reliability block diagram are uniquely determined;
- Each subsystem can be considered consisting of separate subsystems and meets the requirements described above;
- For each element and whole system only two possible states are allowed: workable or failure. Partial functioning of the system or its elements is excluded;
- All random events of the system elements safe operation are independent, i.e. the failure of one element does not affect the performance of other elements

4. An application of the methodology

As an example of the application of the method, consider the system shown in Figure 1. Under reliability analysis select all subsystems. Elements C_5 and C_6 are connected in parallel, they can be represented by subsystem C_{56} , its characteristic is the probability of failure. Elements C_1 , C_2 , C_3 and C_4 , C_{56} are serially connected elements, their characteristics are values of reliability. These elements are represented by the subsystems C_{123} and C_{456} with the corresponding characteristics. In turn, subsystems C_{123} and C_{456} connected parallel represent considered system in general; therefore, the determinant module of the reliability matrix Γ is interpreted as the system probability of failure.

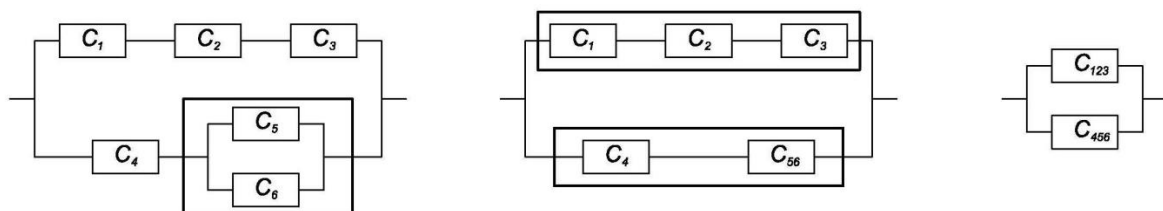


Figure 5. To system reliability analysis

The component characteristic matrix R is expressed by:

$$R = \begin{pmatrix} p_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & p_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & p_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & p_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & q_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & q_6 \end{pmatrix}. \quad (14)$$

Interconnections between elements and subsystems are described above. Form the component connection matrix Ω . Elements C_1, C_2, C_3 connected in series are a subsystem, thence $\omega_{12} = \omega_{23} = -1$, $\omega_{31} = 1$, elements C_5 and C_6 connected in parallel also are a subsystem, thence $\omega_{56} = \omega_{65} = 1$. Element C_4 serially connected with element C_5 and C_6 , therefore $\omega_{45} = -1$, $\omega_{64} = 1$, and elements C_1, C_2, C_3 are parallel connected with C_4 and C_5, C_6 . They are represented system in general that's why $\omega_{34} = 1$. The rest elements of the component connection matrix Ω equal zero, because there is no correlation between corresponding elements

$$\Omega = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}. \quad (15)$$

Thereby reliability matrix Γ is shown as follows:

$$\Gamma = R - \Omega = \begin{pmatrix} p_1 & 1 & 0 & 0 & 0 & 0 \\ 0 & p_2 & 1 & 0 & 0 & 0 \\ -1 & 0 & p_3 & -1 & 0 & 0 \\ 0 & 0 & 0 & p_4 & 1 & 0 \\ 0 & 0 & 0 & 0 & p_5 & -1 \\ 0 & 0 & 0 & -1 & -1 & p_6 \end{pmatrix}. \quad (16)$$

Substituting the values of reliability and probability of failure of the elements the following reliability matrix Γ is occur:

$$\Gamma = \begin{pmatrix} 0.98 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0.98 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0.98 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0.96 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0.01 & -1 \\ 0 & 0 & 0 & -1 & -1 & 0.01 \end{pmatrix}. \quad (17)$$

Then a module of the determinant of the matrix can calculated. Since the system is connected in parallel subsystem, according to this paper the module is the system failure probability.

$$Q_s = |\det(\Gamma)| = 2.358 \cdot 10^{-3}. \quad (18)$$

This result corresponds to the above mentioned one:

$$P_s + Q_s = 0.997642 + 0.002358 = 1. \quad (19)$$

Hence, this algorithm is correct and system reliability matrix method can be used to reliability analysis. As it can be seen in the above example, the computation for the used methodology is very simple and straightforward. It should be noted that to form the matrices, one can use both the reliability block diagram and the Boolean function. The proposed method can be described with program code, used in various software, incl. in computer-aided design systems.

5. Conclusion

Traditional methods of reliability analysis are difficult to apply for complex systems consisting of huge number of elements. The method proposed in this paper contributes to turn from a large calculation to a simple and compact one. It can standardize calculations and be implemented on a computer that will significantly reduce spent time to solve the reliability problem. This method can be used to reliability analysis of a building or structure, the model of which is a complex mechanical system. Solving the problem of reliability of a building object at the design stage with the application of this method allows to quickly comparing several structural system in terms of their reliability, which will increase the level of design in general.

This paper describes a new method for assessing the reliability of mechanical systems, developed on the basis of method reliability analysis using graph theory and Boolean function. The application of the matrix method is illustrated by an example. The probability of failure of an arbitrary system is confirmed by another method of system reliability analysis.

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